EXERCISE 4
MODULAR FORMS 2019
DUE DATE: MAY 12, 2019

Exercise 1. Let $k \geq 12$. Show that the set of Poincaré series

$$\Big\{P_m^k, \quad 1 \leq m \leq \frac{k}{12} + 1\Big\}$$

span the space of cusp forms S_k.

Exercise 2. Let $c \geq 1$ and $m, n \in \mathbb{Z}$ integers. The Kloosterman sum is defined as

$$\mathrm{Kl}(m, n; c) := \sum_{\substack{x \bmod c \\ \gcd(x,c)=1}} e\Big(\frac{mx + n\bar{x}}{c}\Big)$$

where we abbreviate

$$e(z) := e^{2\pi i z}$$

and $\bar{x}$ denotes the multiplicative inverse of x mod c: $x\bar{x} = 1 \bmod c$.

a) Show that if p is prime and $a \neq 0 \bmod p$ then $\mathrm{Kl}(a, 0; p) = -1$.

b) If $a, b \neq 0 \bmod p$ then

$$\mathrm{Kl}(a, b; p) = \mathrm{Kl}(ab, 1; p)$$

Exercise 3. a) Show that the hyperbolic measure $dxdy/y^2$ is invariant under an Mobius transformation $g \in \mathrm{SL}(2, \mathbb{R})$.

b) Compute the hyperbolic area $\int_{\mathcal{F}} \frac{dxdy}{y^2}$ of the standard fundamental domain $\mathcal{F}$ for $\mathrm{SL}(2, \mathbb{Z})$.